Tutorial 3 – Answers

1. Given the set A = {2, 3, 4, 5, 6, 7, 8, 9} form a new set B which consists of all elements of set

A that are:

i Prime numbers.

Prime Numbers - whole numbers greater than 1 that can not be made by multiplying other

whole numbers.

Prime Numbers = {2,3,5,7}

ii Even numbers.

Even Numbers – Numbers that are divisible by 2

Evan numbers = {2,4,6,8}

iii Odd numbers that are greater or equal to 3.

={3,5,7,9}

iv Those numbers that being multiplied by 2 give a number that is also an element of A.

Those numbers that are in set A and also some of its 2s multiples

={2,3,4}

v Those numbers that being multiplied by 2 give a number that is not in the given set A.

Those numbers that are in set A and none of its 2 s multiples are in the set A

={5,6,7,8,9}

vi Those numbers that being squared resulting in a number which also belongs to A.

={2,3}

2. Let N be a set of natural numbers {1, 2, 3 ,….}. For each of the cases below, a new

set B is defined by using set builder notation. List all elements of B and establish the cardinality of it.

i B = {x : x ∈ N and x2 = x} = {1}, Cardinality = 1

x2 = x, x2 – x = 0, x(x-1) = 0, x = 0 or x = 1, but 0 in not in the set

ii B = {x : x ∈N and x2 = 2x} = {2}

x2 = 2x, x2 - 2x = 0,x(x-2) = 0, x =0 or x = 2

iii B = {(x; y) : x ∈ N and y ∈ N and x < y and y ≤ 3}

N = Some definitions, begin the natural numbers with 0, corresponding to the non-negative integers 0, 1, 2, 3, ... , whereas others start with 1, corresponding to the positive integers 1, 2, 3, .... In the questions clearly it will be indicated which definition is used.

={(1,2),(1,3),(2,3)}, cardinality = 3

iv B = {(x; y) : x∈ N and y ∈ N and x = y and y < 5}

={(1,1),(2,2),(3,3),(4,4)} cardinality = 4

v B = {(x; y; z) : x ∈ N and z ∈ N and and x = 10 and 1 < y < 5}

{( 10,2,14),(10,3,18),(10,4,26)}, cardinality = 3

3. Identify if the following statements are true or false.

i. 3 ∈ {3, 4, 5} - T

ii. 3 ∈ {3, 4, 5} - T

iii. {3} ∈ {{3}, {4}, {5}} - T

iv. {3} ⊆ {3,4,5} - T

v. 3 ⊆ { 3,4,5} - F

vi. {] ⊆ {3, 4,5} - T

4. Let A = [2,9] be a closed interval of all real numbers 2 to 9. Which new interval is introduced

by the following set builder notation: {x ∈ A : ∈ A}.

=[4,9]

5. Define, using the set builder notation, the set C which is obtained via the following operations

on sets A and B:

i. C = A ∩ B = {x | x∈ A and x ∈ B}

ii. C = A ∪ B = {x | x ∈ A or x ∈ B

iii. A \ B = {x | x ∈ A and x ∉ B}

iv. = {x } x ∈ U and x ∉ A}

6. Let A and B be the only sets in U and A = {5, 6, 10, 12} and B = {5, 7, 11} Apply the

following operations to sets A and B

i. A ∪ B ={5,6,7,10,11,12} = U

ii. A ∩ B = {5}

iii. A \ B = {6,10,12}

iv. B \ A = {7,11}

v. A’ = {7,11}

vi. B’ = {6,10,12}

vii. A’ ∩ B’ = {}

7. Below U is the universal set, {} is the empty set and A is an arbitrary set. Based on the definition of the empty and universal sets establish what should be the resulting set of the following operations:

i. A ∩ U = A

ii. A ∪ U = U

iii. A \ U = {}

iv. (U \ A) ∪ A = U

v. {} ∪ A = A

vi. {} ∩ A = {}

8. Consider sets A = {a}, B = {g, h, i, j} and C = {i, j, k, l}. In the Venn diagram below place

the elements of the following sets and establish what are the sets resulting in the following

operations:

1. B ∩ C = {i,j}
2. A ∪ B = {a,g,h,i,j}
3. A ∩ B = {}
4. B ∪(B ∩ C) = {g,h,i,j}
5. A ∩(B\U)= {}
6. U ∩ ( A ∪ B) = {a,g,h,i,j}

9. Let A = {a, b, c} and B = {1, 0}.

i. Write down all elements of the Power Set of A and Power Set of B

P(A) = {φ,{a},{b},{c},{a,b},{a,c},{b.c},{a,b,c}}

P(B) = {φ,{1},{0},{1,0}}

ii. List all elements of A x B ={(a,1),(a,0),(b,1),(b,0),(c,1),(c,0)}

iii. List all elements of B x A ={(1,a),(0,a),(1,b),(0,b),(1,c),(0,c)}

iv. Calculate |A x B| and |B x A|= 6 and 6

v. What is (A x B) \ (B x A) = A x B as there are no common elements in Ax B and BxA

10. CHALLENGE

i. Show that if A ⊆ B and B ⊆ A then A = B, i.e. that A and B are equivalent, i.e. they

have the same elements.

Let’s assume that A ⊆ B and B ⊆ A and A # B

Then there should be an element x in A which is not in B

However, A ⊆ B therefor all elements in A should be also in B – Contradiction

Thus A = B

ii. Let A = [2,9] be a closed interval of all real numbers 2 to 9 Which interval is introduced

by the following set builder notation: {x ∈ A : ∈ A and x } = [4,

11. CHALLENGE

Let A = {m : m is an integer satisfying 0 < m < 13} and B = {n : n is an integer satisfying 7 <n < 23}.

Calculate |(A x B) \(B x A)|.

Note that the task is to only calculate the number of elements but not to list the elements of the resulting set!

A = {1,2,3,4,5,6,7,8,9,10,11,12}

B = {8,9,10,……,22}

How many items are common to (A x B) and (B x A) ?

Common elements to both A and B are = {8,9,10,11,12}

Thus, A x B has 12 x 15 = 180 items out of that 5 x5 = 25 items are common

Thus |(A x B) \(B x A)|. = 180 -25 = 155